

The role of coherence during classical and quantum decoherence

Jin-Xing Hou,^{1,2,3,*} Si-Yuan Liu,^{4,2,†} Xiao-Hui Wang,^{3,2} and Wen-Li Yang^{4,2}

¹*Institute of Modern Physics, Northwest University, Xi'an 710069, China*

²*Shaanxi Key Laboratory for Theoretical Physics Frontiers, Xi'an 710069, China*

³*School of Physics, Northwest University, Xi'an 710069, China*

⁴*Institute of Modern Physics, Northwest University, Xi'an 710069, P. R. China*

The total correlations in a bipartite quantum system are measured by the quantum mutual information \mathcal{I} , which consists of quantum discord and classical correlation. However, recent results in quantum information shows that coherence, which is a part of total correlation, is more general and more fundamental. The role of coherence in quantum resource theories is worthwhile to investigate. We study the relation between quantum discord and coherence by decreasing the difference between them. And then, we consider the dynamics of quantum discord, classical correlations and quantum coherence under incoherent quantum channels. It is found that coherence indicate the behavior of quantum discord (classical correlation) for times $t < \bar{t}$, and indicate the decoherence of classical correlation (quantum discord) for times $t > \bar{t}$. What is more, the coherence frozen and decay indicate the quantum discord and classical correlation frozen and decay respectively.

PACS numbers: 03.65, 03.67

I. INTRODUCTION

As a key resources of quantum information process, quantum correlations can be used to achieve different task with different forms, such as entanglement, quantum discord, coherence etc.. It is largely accepted that quantum mutual information is the measure of the total correlation in a bipartite quantum systems, and the total correlation consists of quantum and classical correlation. Aiming to capture the total nonclassical correlations, Ollivier and Zurek proposed a measure called quantum discord [1–3], and stipulated the states with zero quantum discord are called classical correlated states. Quantum discord, as a kind of quantum resource, have been widely studied for past decade [4–17].

Coherence, which marks the departure of quantum theory from classical physics, is closely connected to quantum superposition and quantum correlations. Recently, a rigorous framework have been proposed to quantify coherence [18], and the resource theory of coherence have received a great deal of attention [18–22]. It is notable that, as a kind of quantum resource, coherence have many specific characters. Coherence is a basis-dependent quantifier, and can exist in local subsystem. Coherence can even exist without quantum discord, and thus, coherence is more fundamental than quantum discord. The relation between quantum discord and coherence is important for us to understand the common feature of quantum resources [23–25]. In this paper, we study the relation between quantum discord and coherence by decreasing the difference between them, i.e., relax the basis constraint b and remove the local coherence.

Suppose quantum discord is the quantum part of total correlation, and coherence is a part of total correlation

(quantum mutual information). A nature question is coherence, as a more fundamental quantum resources, is completely quantum or part of quantum and have some classical feature. There are two views about this question: i). Coherence indicates the quantum correlation, discord is a special kind of quantum correlation, like entanglement. ii). Discord indicates the quantum correlation, coherence contains quantum correlation and part of classical correlation. We consider the role of coherence during quantum discord and classical correlation decoherence to answer the question.

The interaction of a quantum system with its environment reveals abundant character of quantum physics, such as frozen [26] and sudden death [27, 28] of quantum resources. We research bipartite system evolution under incoherent quantum channel. We discover that coherence can both indicate the behavior of quantum discord and classical correlation in different times, and the role of coherence changes suddenly at transition time \bar{t} . This phenomenon shows that coherence contains both quantum and classical feature.

The paper is organized as follows. In Sec. II, we review the measures of coherence and quantum discord, and discuss the relationship between coherence and quantum discord measures. In Sec. III, we verify that, for Bell-diagonal states, quantum discord is equal to coherence in optimal basis. In Sec. IV, we study the role of coherence in quantum discord and classical correlation decoherence. In Sec. V, we show that coherence frozen indicates the frozen of quantum discord and classical correlation, and coherence decay indicates the decay of quantum discord and classical correlation. We summarize our conclusion and future research in Sec. VI.

* jinxhou@163.com

† syliu@iphy.ac.cn

II. MEASURES OF QUANTUM CORRELATION

A. Measures of coherence

A reasonable measure to quantify coherence should fulfill [18]: Nonnegativity, $\mathcal{C}(\rho) \geq 0$ with equality if and only if ρ is incoherent; Monotonicity, \mathcal{C} do not increase under the action of incoherent operations, $\mathcal{C}(\Lambda[\rho]) \leq \mathcal{C}(\rho)$, for any incoherent operation Λ ; Strong monotonicity, \mathcal{C} do not increase on average under selective incoherence operations, $\sum_i q_i \mathcal{C}(\sigma_i) \leq \mathcal{C}(\rho)$, with probabilities $q_i = \text{Tr}[K_i \rho K_i^\dagger]$, $\sigma_i = K_i \rho K_i^\dagger / q_i$, and incoherent Kraus operators K_i ; Convexity, \mathcal{C} is a convex function of the state, $\sum_i p_i \mathcal{C}(\rho_i) \geq \mathcal{C}(\sum_i p_i \rho_i)$. In accordance with the set of properties which every proper measure of coherence should satisfy, a number of coherence measure have been put forward. We focus on the relative entropy of coherence and l_1 norm of coherence. The relative entropy of coherence is defined as

$$\mathcal{C}_r(\rho) = \min_{\delta \in \mathcal{I}} S(\rho \| \delta) = S(\rho_{diag}) - S(\rho), \quad (1)$$

where ρ_{diag} comes from ρ by vanishing off-diagonal elements, $S(\rho \| \delta) = \text{Tr}(\rho \log \rho - \rho \log \delta)$ is the quantum relative entropy and $S(\rho) = -\text{Tr}(\rho \log \rho)$ is the von Neumann entropy [29]. The l_1 norm of coherence is defined

$$\mathcal{C}_{l_1}(\rho) = \min_{\delta \in \mathcal{I}} |\rho - \delta|_{l_1} = \sum_{i \neq j} |\rho_{ij}|, \quad (2)$$

where ρ_{ij} are entries of ρ .

B. Measures of quantum discord

The quantum mutual information of system A and B is defined as

$$\mathcal{I}(\rho_{AB}) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}). \quad (3)$$

The one-side classical mutual information is given by the following form

$$\mathcal{J}_{cq}(\rho_{A:B}) = S(B) - S(B|\{E_a\}), \quad (4)$$

where $S(B|\{E_a\}) = \sum_a p_a S(\rho_{B|a})$ is the conditional entropy [29], and $\{E_a\}$ is a set of POVM measurement with elements $E_a = M_a^\dagger M_a$ and classical outcome a on subsystem A . The minimized difference between quantum mutual information and one-side classical mutual information

$$\mathcal{D}_1(\rho_{AB}) \equiv \min_{\{E_a\}} \{\mathcal{I}(\rho_{A:B}) - \mathcal{J}_{cq}(\rho_{A:B})\}, \quad (5)$$

was called quantum discord by Olliver and Zurek [1]. The classical correlation was proposed by Henderson and Vedral [2]

$$\mathcal{CC}_{cq}(\rho_{AB}) \equiv \max_{E_a} \mathcal{J}_{cq}(\rho_{A:B}), \quad (6)$$

where the maximum is taken over the complete set of positive operator-valued measure (POVM) $\{E_a\}$. Then quantum discord is simply defined as $D(\rho_{AB}) = \mathcal{I} - \mathcal{CC}(\rho_{AB})$.

We can also define quantum and classical correlations via two-side measurement. The two-side classical correlation in a composite bipartite system can be expressed as the maximum classical mutual information

$$\mathcal{CC}_{cc}(\rho_{AB}) \equiv \max_{E_a \otimes E_b} \mathcal{I}_c(\rho_{A:B}), \quad (7)$$

where $\mathcal{I}_c(\rho_{A:B}) = \mathcal{H}(\rho_A) + \mathcal{H}(\rho_B) - \mathcal{H}(\rho_{AB})$ is classical mutual information and $\mathcal{H} = -\sum_i p_i \log p_i$ is Shannon entropy. The two-side quantum discord is defined as

$$\mathcal{D}_2(\rho_{AB}) = \mathcal{I}(\rho_{AB}) - \mathcal{CC}_{cc}(\rho_{AB}). \quad (8)$$

The relative entropy of discord is defined as

$$\mathcal{D}_r(\rho_{AB}) = \min_{\delta} S(\rho_{AB} \| \chi) = \min_{\mathcal{B}(\vec{k})} \mathcal{H}(\mathcal{B}(\vec{k})) - S(\rho_{AB}), \quad (9)$$

where χ is in the set of classical-classical states and $\{|\mathcal{B}(\vec{k})\rangle\} = |\mathcal{B}(k_1)\rangle |\mathcal{B}(k_2)\rangle$ is a local orthogonal basis.

C. Relation of quantum discord and coherence

Coherence and discord in a given state is the distance to the closest incoherent state and classical correlated states. The relative entropy of quantum discord and relative entropy of coherence are defined as [18, 31]

$$\mathcal{D} = \min_{\delta \in \mathcal{CC}} S(\rho \| \delta), \quad (10)$$

$$\mathcal{C} = \min_{\delta \in \mathcal{I}} S(\rho \| \delta), \quad (11)$$

where \mathcal{CC} and \mathcal{I} stand for the sets of classically correlated states and incoherent states. The classically correlated states and incoherent states have the form

$$\delta = \sum_k p_k \tau_{k,1}^{(b)} \otimes \cdots \otimes \tau_{k,n}^{(b)}, \quad (12)$$

where $|b_{k,n}\rangle$ and $\tau_{k,n}^{(b)} = \sum_k p_{k,n} |b_{k,n}\rangle \langle b_{k,n}|$ are a fixed particular basis b and incoherent state on the subsystem n respectively. Coherence is a basis-dependent quantity, different basis generate different coherence. Whereas, quantum discord is not a basis-depend quantity. We attempt to give an alternative understanding of how quantum discord is related to quantum coherence. To reduce the difference between quantum discord and coherence, we relax the basis constraint b and minimize over the set of measurement basis. Denote the basis b^{opt} is the minimum solution of quantum discord, we define relative entropy of coherence in basis b^{opt} as \mathcal{C}^{opt} . The relation between quantum discord and coherence can be expressed as follows

Theorem 1 *The relative entropy of quantum discord is equal to relative entropy of coherence in an optimal basis.*

Proof:

$$\begin{aligned}\mathcal{D}_r(\rho_{AB}) &= \min_{\mathcal{B}(\vec{k})} \{ \mathcal{H}(\mathcal{B}(\vec{k})) - S(\rho_{AB}) \} \\ &= \min_{\mathcal{B}} \{ S(\rho_{AB}^{diag}) - S(\rho_{AB}) \} \\ &= \mathcal{C}_r(\rho_{AB})^{opt}. \end{aligned} \quad (13)$$

Quantum discord exist at least in two-partite system, but coherence can even exist in one-partite system. We may remove the local coherence and choose an optimal basis, and then, the relation between two-side quantum discord and relative entropy of coherence can be expressed as follows

Theorem 2 *Two-side quantum discord is equal to relative entropy of coherence between subsystem A and B in an optimal basis.*

Proof:

$$\begin{aligned}\mathcal{D}_2(\rho_{AB}) &= \min_{\Pi_i} \{ \mathcal{I}(\rho_{AB}) - \mathcal{I}_{cc}(\rho_{AB}) \} \\ &= \min_{\Pi_i} \{ \mathcal{C}_r(\rho_{AB}) - \mathcal{C}_r(\rho_A) - \mathcal{C}_r(\rho_B) \} \\ &= \mathcal{C}_r(\rho_{AB})^{opt} - \mathcal{C}_r(\rho_A)^{opt} - \mathcal{C}_r(\rho_B)^{opt}. \end{aligned} \quad (14)$$

III. COHERENCE AND QUANTUM DISCORD FOR TWO-QUBIT SYSTEM

The most general two-qubit states ρ_{AB} can be expressed as [32]

$$\rho_{AB} = \frac{1}{4}(I \otimes I + \vec{a}\vec{\sigma} \otimes I + I \otimes \vec{b}\vec{\sigma} + \sum_{i,j=1}^3 c_{ij}\sigma_i \otimes \sigma_j), \quad (15)$$

Here I is the identity, $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ with $\sigma_1, \sigma_2, \sigma_3$ being the Pauli operator. \vec{a}, \vec{b} are vectors in \mathbb{R}^3 , and $c_{i,j}$ are real number. For simplicity, we will only consider the Bell-diagonal states [33], which can be parameterized as

$$\rho_{AB} = \frac{1}{4}(I \otimes I + \sum_{j=1}^3 c_j \sigma_j \otimes \sigma_j) = \sum_{ab} \lambda_{ab} |\beta_{ab}\rangle \langle \beta_{ab}|, \quad (16)$$

with the maximally mixed marginals ($\rho_A = \rho_B = \frac{I}{2}$). The density matrix of Bell-diagonal states with σ_3 representation takes the form

$$\rho_{AB}^{\sigma_z} = \frac{1}{4} \begin{pmatrix} 1+c_3 & 0 & 0 & c_1-c_2 \\ 0 & 1-c_3 & c_1+c_2 & 0 \\ 0 & c_1+c_2 & 1-c_3 & 0 \\ c_1-c_2 & 0 & 0 & 1+c_3 \end{pmatrix}, \quad (17)$$

The eigenstates of $\rho_{AB}^{\sigma_z}$ are the four Bell states:

$$|\beta_{ab}\rangle = (|0, b\rangle + (-1)^a |1, 1 \oplus b\rangle) / \sqrt{2}, \quad (18)$$

and the corresponding eigenvalues are

$$\lambda_{ab} = \frac{1}{4}[1 + (-1)^a c_1 - (-1)^{a+b} c_2 + (-1)^b c_3], \quad (19)$$

where $a, b \in \{0, 1\}$. The Bell-diagonal states are a three-parameter set, whose geometry can be depicted as a tetrahedron \mathcal{T} in three-parameter space, see Fig. 1.

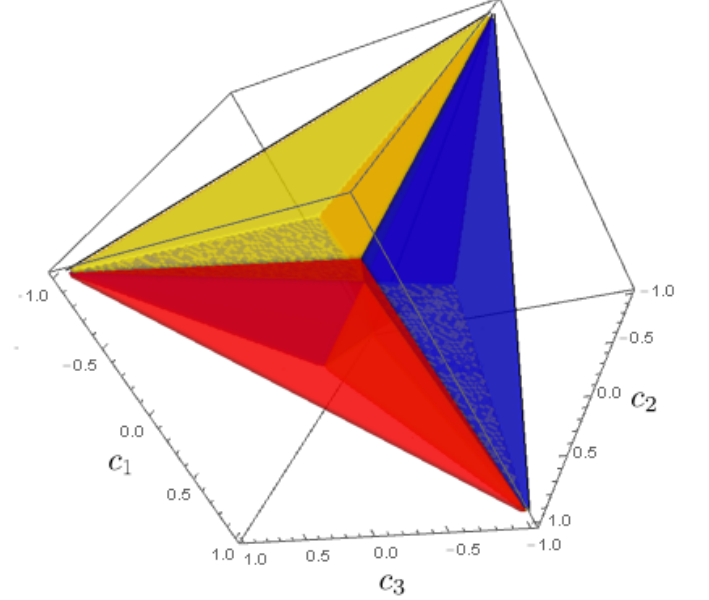


Figure 1. According to quantum discord and classical correlation, Bell-diagonal states in the tetrahedron can be divided into three reigns. The sudden change of quantum discord and classical correlation appears in the contact area of those three area. In yellow reign, $c = c_1$ and $\mathcal{D}(\rho_{AB}) = \mathcal{C}(\rho_{AB})^{\sigma_1}$. In red reign, $c = c_2$ and $\mathcal{D}(\rho_{AB}) = \mathcal{C}(\rho_{AB})^{\sigma_2}$. In blue reign, $c = c_3$ and $\mathcal{D}(\rho_{AB}) = \mathcal{C}(\rho_{AB})^{\sigma_3}$. As for Werner states, $\mathcal{C}_r(\rho_{AB}^{\sigma_1}) = \mathcal{C}_r(\rho_{AB}^{\sigma_2}) = \mathcal{C}_r(\rho_{AB}^{\sigma_3}) = \mathcal{D}(\rho_{AB})$.

Recall that two orthonormal basis sets A and B , for a d -dimensional Hilbert space, are said to be mutually unbiased base, or maximally complementary, if their overlaps are constant, if $|\langle A|B \rangle| = d^{-1}$ for all a and b . Three Pauli qubit observables σ_1 , σ_2 , and σ_3 are mutually unbiased, in the sense that the distribution of any one of these observables is uniform for any eigenstate of the others [34–37]. The Bell-diagonal states with σ_x and σ_y representation take the forms of

$$\rho_{AB}^{\sigma_1} = \frac{1}{4} \begin{pmatrix} 1+c_1 & 0 & 0 & c_3-c_2 \\ 0 & 1-c_1 & c_3+c_2 & 0 \\ 0 & c_3+c_2 & 1-c_1 & 0 \\ c_3-c_2 & 0 & 0 & 1+c_1 \end{pmatrix}, \quad (20)$$

and

$$\rho_{AB}^{\sigma_2} = \frac{1}{4} \begin{pmatrix} 1+c_2 & 0 & 0 & c_3-c_1 \\ 0 & 1-c_2 & c_1+c_3 & 0 \\ 0 & c_1+c_3 & 1-c_2 & 0 \\ c_3-c_1 & 0 & 0 & 1+c_2 \end{pmatrix}. \quad (21)$$

For Bell-diagonal states, there are no coherence in subsystem. The distribution of coherence is simple and just between two subsystem. The relative entropy of coherence is given by

$$\mathcal{C}_r(\rho_{AB}^{\sigma_i}) = -H(\lambda_{ab}) - \sum_{j=1}^2 \frac{(1 + (-1)^j c_i)}{2} \log_2 \frac{(1 + (-1)^j c_i)}{4}, \quad (22)$$

where $H(\lambda_{ab}) = -\sum_{ab} \lambda_{a,b} \log_2 \lambda_{a,b}$. The l_1 norm of coherence is

$$\mathcal{C}_{l_1} = \frac{1}{2}|c_1 - c_2| + \frac{1}{2}|c_1 + c_2|. \quad (23)$$

The mutual information for Bell-diagonal states is given by

$$\mathcal{I} = \sum_{a,b} \lambda_{ab} \log_2(4\lambda_{ab}). \quad (24)$$

The classical correlation for Bell-diagonal states is given by

$$\mathcal{CC} = \sum_{j=1}^2 \frac{(1 + (-1)^j c)}{2} \log_2(1 + (-1)^j c), \quad (25)$$

where $c = \max\{|c_1|, |c_2|, |c_3|\}$. The quantum discord for Bell-diagonal states is given by [16]

$$\mathcal{D}(\rho_{AB}) = -H(\lambda_{ab}) - \sum_{j=1}^2 \frac{(1 + (-1)^j c)}{2} \log_2 \frac{(1 + (-1)^j c)}{4}. \quad (26)$$

We note that one-side quantum discord, two-side quantum discord and relative entropy of quantum discord are identical for Bell-diagonal states. It is easy to verify that quantum discord is equal to coherence with an optimal basis, and the sudden change of optimal basis comes from the sudden change of classical correlation. We show that the relation between quantum discord and coherence intuitively in Tab I.

Table I. As for Bell-diagonal states, quantum discord is equal to coherence with optimal basis.

region	$c = c_1$	$c = c_2$	$c = c_3$
quantum discord	$\mathcal{C}_r(\rho_{AB}^{\sigma_1})$	$\mathcal{C}_r(\rho_{AB}^{\sigma_2})$	$\mathcal{C}_r(\rho_{AB}^{\sigma_3})$

IV. CORRELATIONS DECOHERENCE

If no instructions, we choose σ_z representation for Bell-diagonal states in this manuscript. In this section, we study the quantum discord, classical correlation and coherence decoherence under phase flip channel. Phase flip

channel has operation elements [29]

$$\begin{aligned} K_{20} &= \sqrt{1 - q(t)/2}I, \\ K_{21} &= \sqrt{q(t)/2}\sigma_2. \end{aligned} \quad (27)$$

where $q = e^{-2\gamma t}$ with damping rate γ is noisy strength. We put phase flip channel on system A and system B respectively, the time evolution of Bell-diagonal states can be expressed as

$$\begin{aligned} c_1(t) &= c_1(0)e^{-2\gamma t}, \\ c_2(t) &= c_2(0)e^{-2\gamma t}, \\ c_3(t) &\equiv c_3(0). \end{aligned} \quad (28)$$

For the states $c_3 = \max\{|c_1|, |c_2|, |c_3|\}$, $\mathcal{C}_r(\rho_{AB}) = \mathcal{D}(\rho_{AB})$, For the states $c_1 = \max\{|c_1|, |c_2|, |c_3|\}$, $\mathcal{C}_{re}(\rho_{AB}) > \mathcal{D}(\rho_{AB})$, $\mathcal{CC}(\rho_{AB}) = H(\mathcal{C}_{l_1})$ for the times $t < \bar{t}_1 = -\frac{\ln c_3(0) - \ln c_1(0)}{2\gamma}$, and $\mathcal{C}_r(\rho_{AB}) = \mathcal{D}(\rho_{AB})$ for the times $t > \bar{t}_1 = -\frac{\ln c_3(0) - \ln c_1(0)}{2\gamma}$. For the states $c_2 = \max\{|c_1|, |c_2|, |c_3|\}$, $\mathcal{C}_{re}(\rho_{AB}) > \mathcal{D}(\rho_{AB})$, $\mathcal{CC}(\rho_{AB}) = H(\mathcal{C}_{l_1})$ for the times $t < \bar{t}_2 = -\frac{\ln c_3(0) - \ln c_2(0)}{2\gamma}$, and $\mathcal{C}_r(\rho_{AB}) = \mathcal{D}(\rho_{AB})$ for the times $t > \bar{t}_2 = -\frac{\ln c_3(0) - \ln c_2(0)}{2\gamma}$. As c_3 close to zero, \bar{t} is increasing exponentially. While $c_3 \rightarrow 0$, $\bar{t} \rightarrow \infty$, coherence describe the behavior of classical correlation all the time. Coherence describes the behavior of classical correlation for times $t < \bar{t}$, but coherence describe the behavior of quantum discord for times $t > \bar{t}$. The sudden change also comes from the sudden change of quantum discord and classical correlation. The trajectory of time evolution from red or yellow reign to blue reign at \bar{t} , and the time \bar{t} is the time where the role of coherence sudden change. In Tab. II, We show the relation of quantum discord, classical correlation and coherence in different times.

Table II. Coherence indicates quantum discord and classical correlation decoherence in different times.

times	$t < \bar{t}$	$t > \bar{t}$
classical correlation	$H(\mathcal{C}_{l_1})$	$\mathcal{I} - \mathcal{C}_r$
quantum discord	$\mathcal{I} - H(\mathcal{C}_{l_1})$	\mathcal{C}_r

The fact that coherence describe the behavior of quantum discord and classical correlation in different times also exist in σ_1 and σ_2 representation, just replace phase flip channel by bit flip and bit-phase flip channel.

V. THE BEHAVIOR OF CORRELATIONS INDICATED BY COHERENCE

Sudden transition between classical correlation and quantum discord loss in a composite system has been studied in [9]. We are going to study the role of coherence in sudden transition between classical correlation and quantum discord loss.

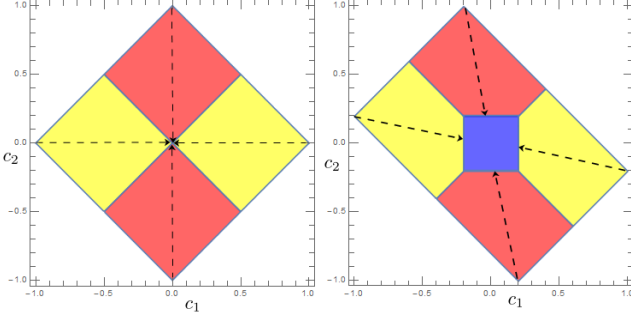


Figure 2. The left is the trajectory of states with $c_3 = 0$, and the right is for the states with $c_3 = 0.2$. In blue reign, $c = c_3$ and $\mathcal{D}(\rho_{AB}) = \mathcal{C}(\rho_{AB})^{\sigma_3}$, and out of blue reign $\mathcal{C}\mathcal{C} = c_{l_1}$. The trajectory of time evolution of Bell-diagonal states under phase flip channel is close to c_3 axis along straight line. The trajectory of time evolution for Bell-diagonal states will get to blue reign over time \bar{t} , and the role of coherence sudden change at time \bar{t} .

A. Coherence indicates the frozen of quantum discord and classical correlation

The *bit flip* channel flip the state of qubit from $|0\rangle$ to $|1\rangle$ (and vice versa) with noisy strength q . It has operation elements [29]

$$\begin{aligned} K_{10} &= \sqrt{1 - q(t)/2} I \\ K_{11} &= \sqrt{q(t)/2} \sigma_1. \end{aligned} \quad (29)$$

We put bit flip channel on system A and system B respectively, the time evolution of Bell-diagonal states can be expressed as

$$\begin{aligned} c_1(t) &\equiv c_1(0), \\ c_2(t) &= c_2(0)e^{-2\gamma t}, \\ c_3(t) &= c_3(0)e^{-2\gamma t}. \end{aligned} \quad (30)$$

For the initial states $c_2(0) = -c_1(0)c_3(0)$, the coherence is frozen [26]. For the initial states $c_2(0) = -c_1(0)c_3(0)$, $c \neq c_1$ the quantum discord is frozen but the classical correlation is decreasing before the transition time \bar{t} , and the quantum discord is decreasing but the classical correlation is frozen over the transition time \bar{t} . For the initial states $c_2(0) = -c_1(0)c_3(0)$, $c = c_1$, coherence shows the frozen of classical correlation for all time. Coherence shows the frozen phenomenon of quantum discord and classical correlation. In Fig. 3, we plot the time evolution of the quantum discord, the classical correlations, and the coherence for $c_1 = 0.6$, $c_2(0) = -0.6$, $c_1(0) = 1$ and $\gamma = 0.1$. The plot clearly shows the frozen of coherence indicates the frozen of quantum discord and classical correlation in different times. The action of bit-phase flip channel on initial states $c_2(0) = -c_1(0)c_3(0)$ also indicated that coherence shows the frozen phenomenon of quantum discord and classical correlation.

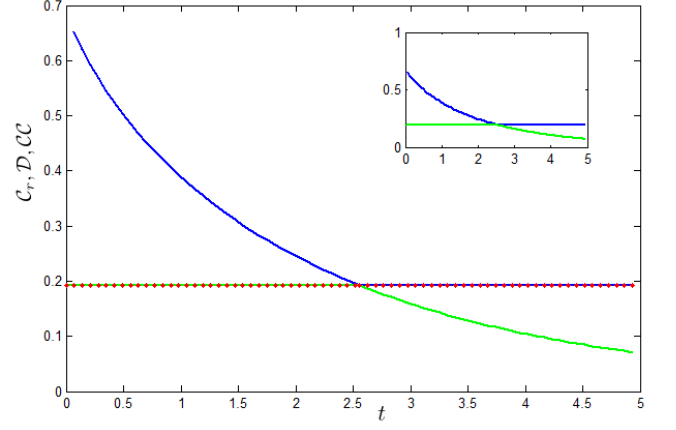


Figure 3. For the initial states $c_1 = 0.6$, $c_2(0) = -0.6$, $c_1(0) = 1$ and $\gamma = 0.1$, coherence (red dots) shows the frozen phenomenon of quantum discord (green line) and classical correlation (blue line) under bit flip channel.

B. Coherence indicates the decay of quantum discord and classical correlation

The *phase flip* channel has operation elements [29]

$$\begin{aligned} K_{20} &= \sqrt{1 - q(t)/2} I, \\ K_{21} &= \sqrt{q(t)/2} \sigma_2. \end{aligned} \quad (31)$$

We put phase flip channel on system A and system B respectively, the time evolution of Bell-diagonal states can be expressed as

$$\begin{aligned} c_1(t) &= c_1(0)e^{-2\gamma t}, \\ c_2(t) &= c_2(0)e^{-2\gamma t}, \\ c_3(t) &\equiv c_3(0). \end{aligned} \quad (32)$$

For the initial states $c_2(0) = -c_1(0)c_3(0)$, $c \neq c_3$, coherence shows the decay of classical correlation for times $t < \bar{t}$, and shows the quantum discord decay for times $t > \bar{t}$. For the initial states $c_2(0) = -c_1(0)c_3(0)$ and $c = c_3$, and coherence shows the decay of quantum discord for all times. In Fig. 4, we plot the time evolution of the quantum discord, the classical correlations, and the coherence for $c_1 = 1$, $c_2(0) = -0.6$, $c_1(0) = 0.6$ and $\gamma = 0.1$. The plot clearly shows the decay of coherence indicates the decay of quantum discord and classical correlation in different times. The action of phase damping channel on initial states also indicated that coherence shows the decay of quantum discord and classical correlation.

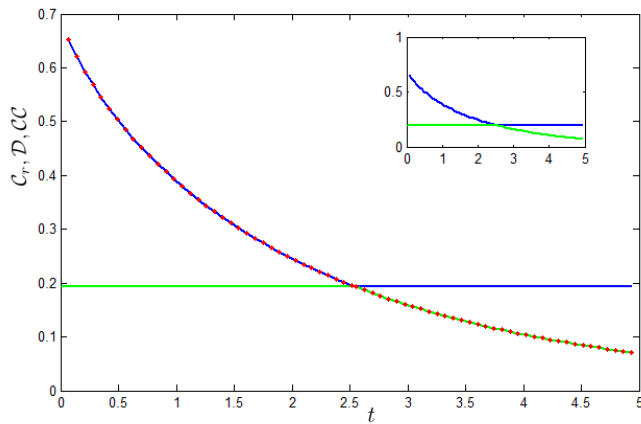


Figure 4. For the initial states $c_1 = 1$, $c_2(0) = -0.6$, $c_1(0) = 0.6$ and $\gamma = 0.1$, coherence (red dots) shows the decreasing phenomenon of quantum discord (green line) and classical correlation (blue line).

VI. CONCLUSION

The total correlation in bipartite system is measured by quantum mutual information. The quantum mutual information consists of quantum discord and classical correlation. Coherence, as a part of quantum mutual information, is different from quantum discord. In this paper, we consider the role of coherence in time evolution of bipartite system.

We study the relationship between quantum discord

and coherence. The relative entropy of quantum discord is equal to relative entropy of coherence in optimal basis. The two-side quantum discord is equal to the total coherence move out the local coherence in optimal basis. For Bell-diagonal states, local coherence is zero, the total coherence is only distributed in the coherence between two bodies.

It is shown that quantum discord is a kind of coherence. Coherence can both describe the behavior of quantum discord and classical correlation. Coherence frozen indicates the quantum discord and classical correlation frozen in different times. Coherence decay also indicates the quantum discord and classical correlation decay in different times.

Our work is important to understand the common feature of quantum resources and the role of quantum discord, classical correlation and coherence in quantum resources theories. We will study the essential relation between quantum resources in the future.

ACKNOWLEDGMENTS

We thank Feng-Lin Wu and Hai-Long Shi for their valuable discussions. This work was supported by the NSFC (Grant No.11375141 and No.11425522), the Special Research Funds of Shaanxi Province Department of Education (No.203010005), Northwest University Scientific Research Funds (No.338020004) and the Double First-Class University Construction Project of Northwest University.

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